

# Work, Energy and Power



Juffrou Karen

# Correct symbols and units

Work      W      J (N.m)

Energy      E      J

Power      P      J.s



# Work done on an object ...

Define the work done on an object by a constant force  $F$  as  $F \Delta x \cos\theta$ , where  $F$  is the **magnitude** of the force,  $\Delta x$  the **magnitude** of the displacement and  $\theta$  the angle between the force and the displacement.



# Power...

Define power as ...

The rate at which work is done OR energy is expended.



# Energy (E)

The ability to do work



# Potential energy ( $E_p$ )

The energy an object has due to its position/height above the earth

$$E_p = mgh$$



# Kinetic energy ( $E_k$ )

The energy an object has due to its motion

$$E_k = \frac{1}{2} mv^2$$



# Mechanical energy

Mechanical energy is the sum of the potential and kinetic energy at a specific point.

$$E_M = E_p + E_k$$



**Work** is the product of force and displacement, with the force and the displacement in the same straight line.

$$W = F \Delta x \cos\theta$$

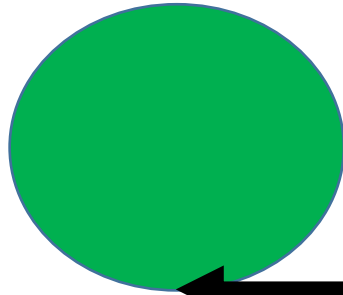


$$W = F\Delta x \cos\theta$$

$$\text{Joule (J)} = \text{N}\cdot\text{m}$$

Work is a scalar  
quantity



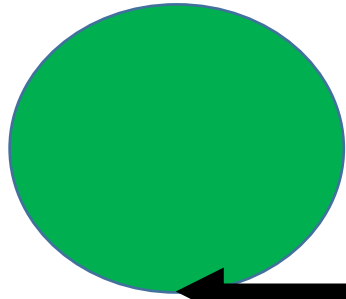


$\Delta x$



$$W = F \Delta x \cos\theta$$

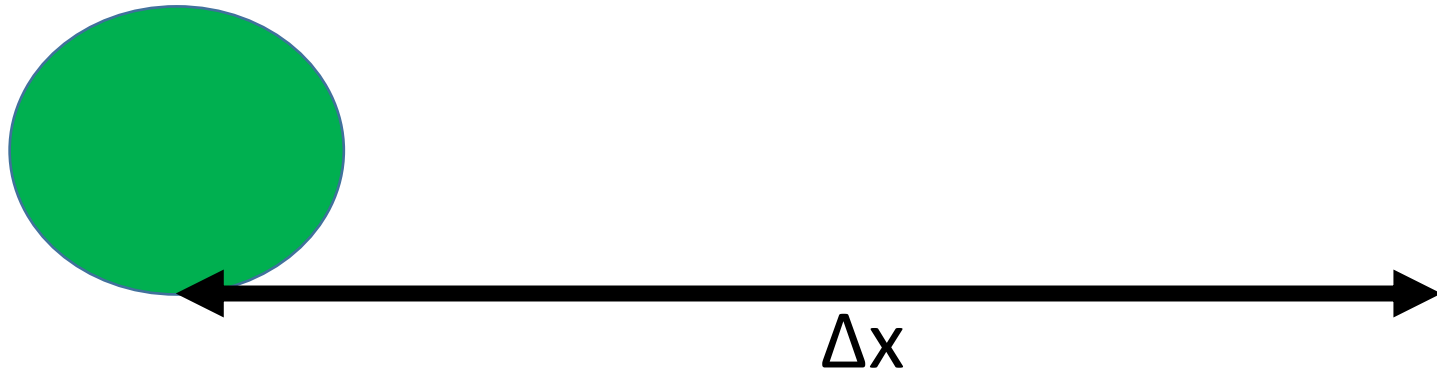
$$\text{Where } \cos(0^\circ) = 1$$



$$W = F \Delta x \cos\theta = 0 \text{ J}$$

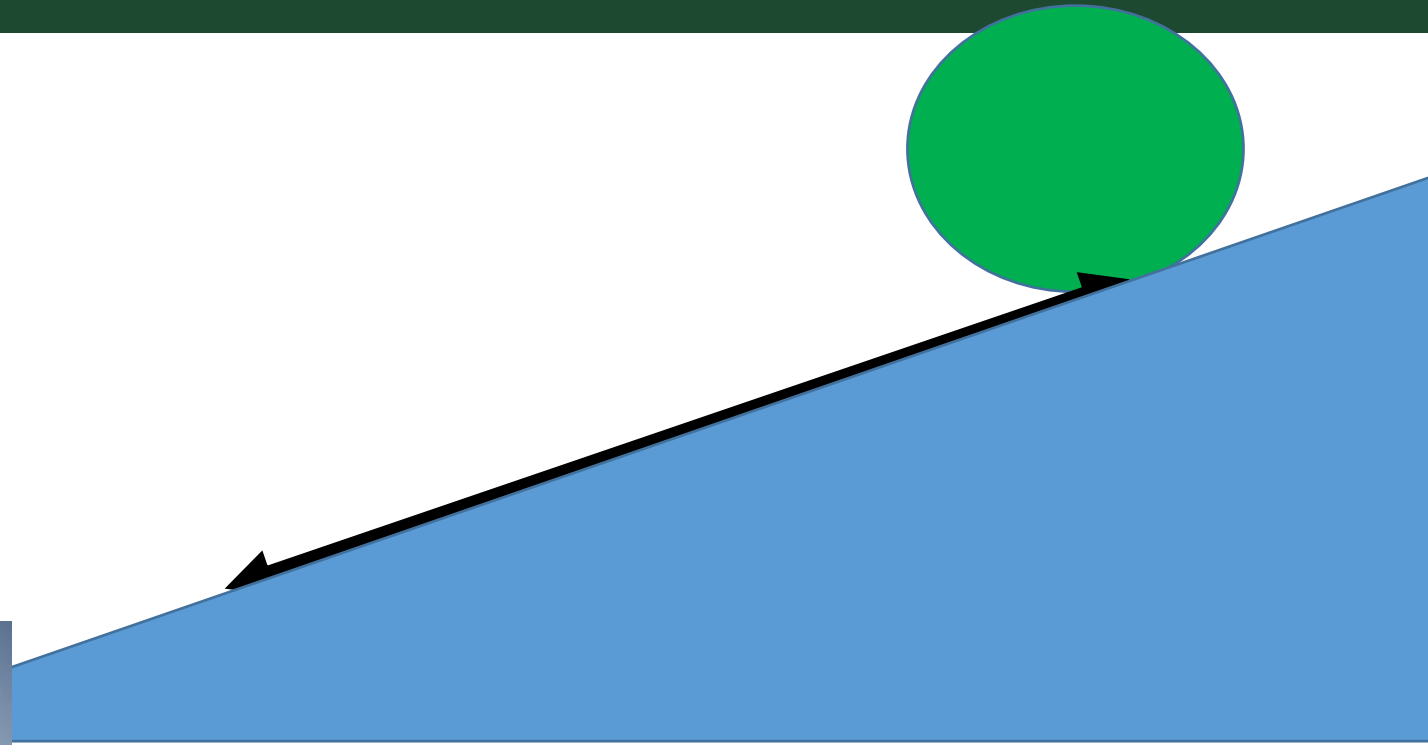
$$\text{Where } \cos(90^\circ) = 0$$





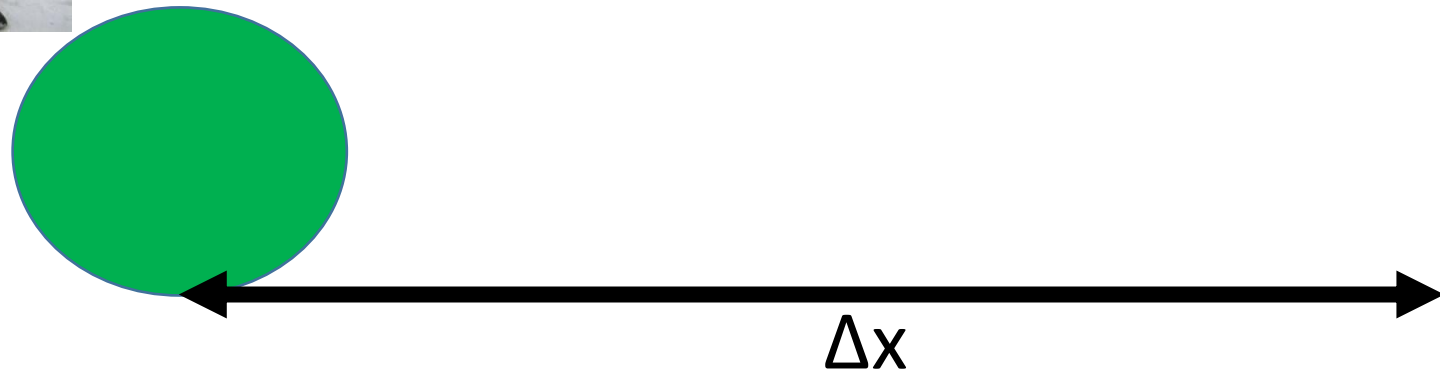
$$W = F \Delta x \cos\theta = -F\Delta x$$

Where  $\cos(180^\circ) = -1$



$$W_{F_g} = F_{g\parallel} \Delta x \cos\theta$$

Where  $\cos(0^\circ) = 1$



$$W_{\text{net}} = (F_x - f_k) \Delta x \cos\theta$$

$$\text{Where } \cos(0^\circ) = 1$$

Negative work  $\rightarrow$  total energy of a system decreases.

Therefore the brakes of a car will decrease the total energy of the system and negative work is performed.

# Example 1

A book is pushed over a distance of 1,5m on a horizontal table top with a horizontal force of 2N. The frictional force is 0,4N.

1.1 Calculate the work done by the 2N force onto the book.

$$\begin{aligned}W_F &= F \Delta x \cos\theta \\ &= 2 \times 1,5 \times \cos(0^\circ) \\ &= 3\text{J}\end{aligned}$$

# Example 1

A Book is pushed over a distance of 1,5m on a horizontal table top with a horizontal force of 2N. The frictional force is 0,4N.

1.2 Calculate the work done by the frictional force onto the book.

$$\begin{aligned}W_f &= f \Delta x \cos\theta \\ &= 0,4 \times 1,5 \times \cos(180^\circ) \\ &= -0,6\text{J}\end{aligned}$$

# Example 1

A Book is pushed over a distance of 1,5m on a horizontal table top with a horizontal force of 2N. The frictional force is 0,4N.

1.3 Calculate the net work done onto the book by the two forces.

$$\begin{aligned}W_{\text{net}} &= F_{\text{net}} \Delta x \cos\theta \\ &= 1,6 \times 1,5 \times \cos(0^\circ) \\ &= 2,4\text{J}\end{aligned}$$

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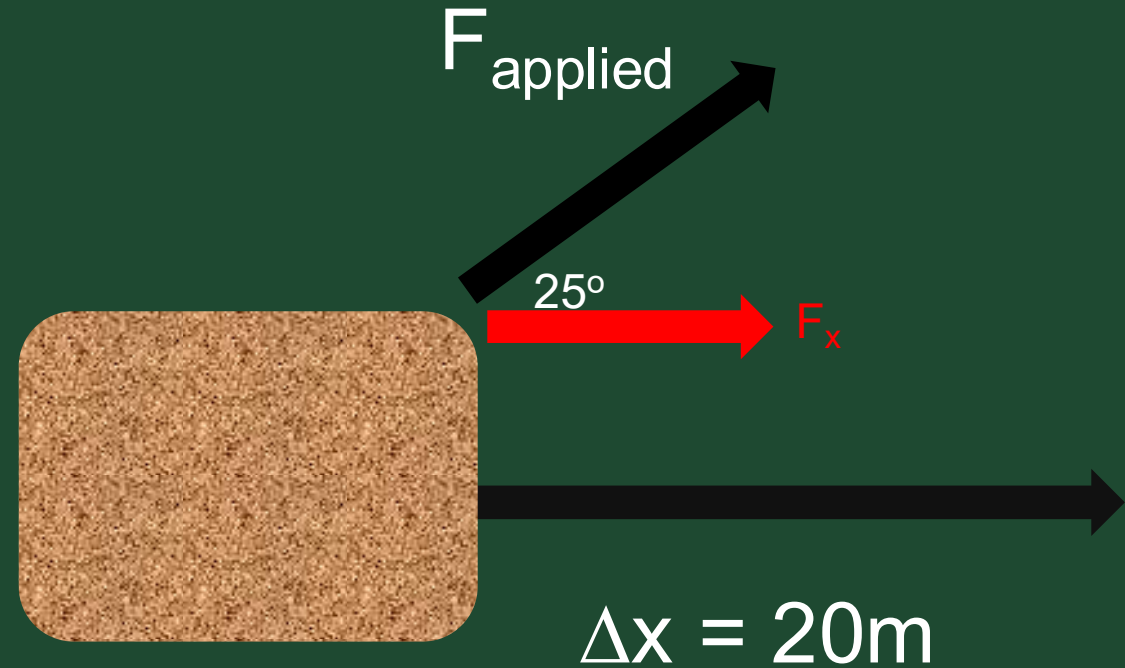
1.4 Is any work done by the normal force and the gravitational force onto the book? Explain your answer.

No. The book is not displaced in the directions of the gravitational/normal forces.

$$W_{F_g} = F_g \Delta x \cos 90^\circ \text{ and } W_N = N \Delta x \cos 90^\circ$$

# Example 2

A factory worker pulls a loaded trolley across a horizontal floor. He applies a force of **750N** to the handle of the trolley, which forms an angle of  $25^\circ$  to the floor. The trolley experiences a frictional force of 250N when it moves 20m over the floor.

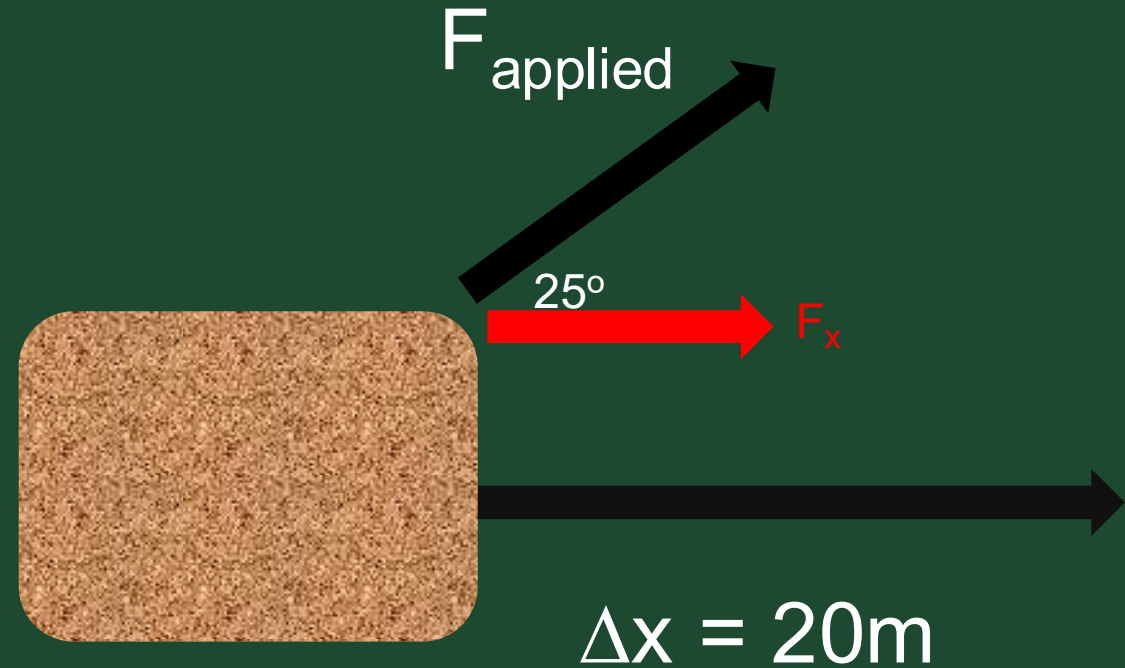


2.1 Calculate the work done by the applied force to the trolley.

$$\begin{aligned} W_F &= F_x \Delta x \cos\theta \\ &= (750\cos 25^\circ) (20) \cos(0^\circ) \\ &= 13594,62\text{J} \end{aligned}$$

# Example 2

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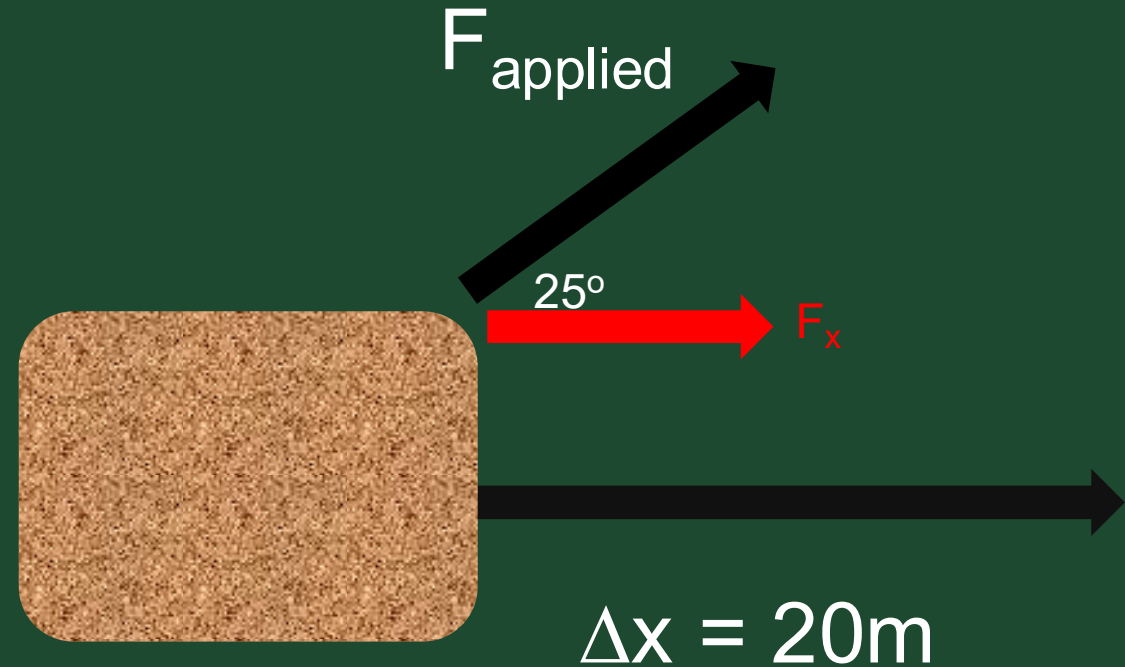
2.2 Calculate the work done by the friction.

$$\begin{aligned}W_f &= F_f \Delta x \cos\theta \\&= 250 \times 20 \cos(180^\circ) \\&= -5000\text{J}\end{aligned}$$

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2.3 Calculate the work done by the weight of the trolley.



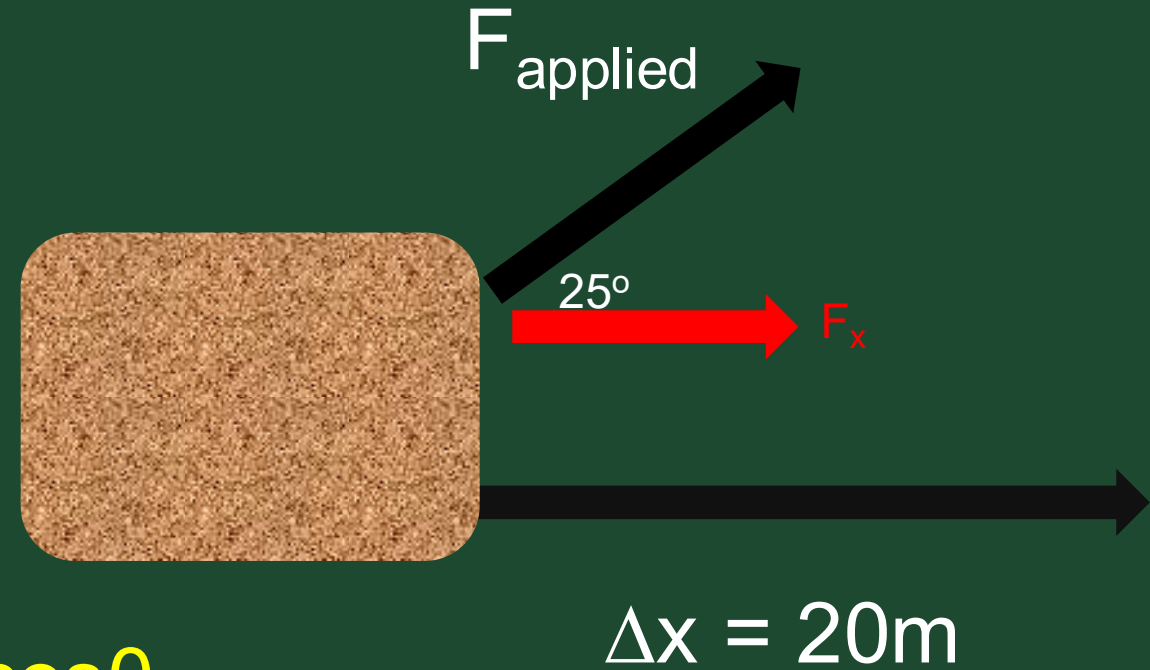
$$\begin{aligned}W_{F_g} &= F_g \Delta x \cos\theta \\ &= mg \times 20 \cos(90^\circ) \\ &= 0\text{J}\end{aligned}$$

There is no component of gravity in the same direction as the movement

# Example 2

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2.4 Calculate the work done by the normal force.



$$\begin{aligned}W_N &= N \Delta x \cos\theta \\ &= mg \times 20 \cos(90^\circ) \\ &= 0\text{J}\end{aligned}$$

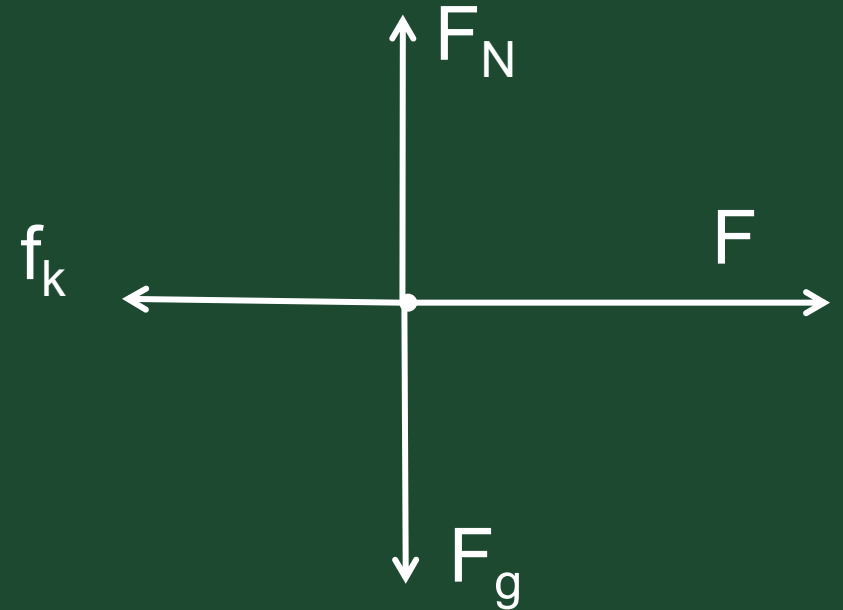
There is no component of the normal force in the same direction as the movement

# Example 3

A learner tries to move a heavy cabinet, with a mass of 80kg, by applying a 50N push force parallel to the floor on it. The cabinet experiences a frictional force of 39,2N. He manages to move the cabinet for a distance of 5m.



- 3.1 Draw a free-body diagram and show all the forces acting on the cabinet.

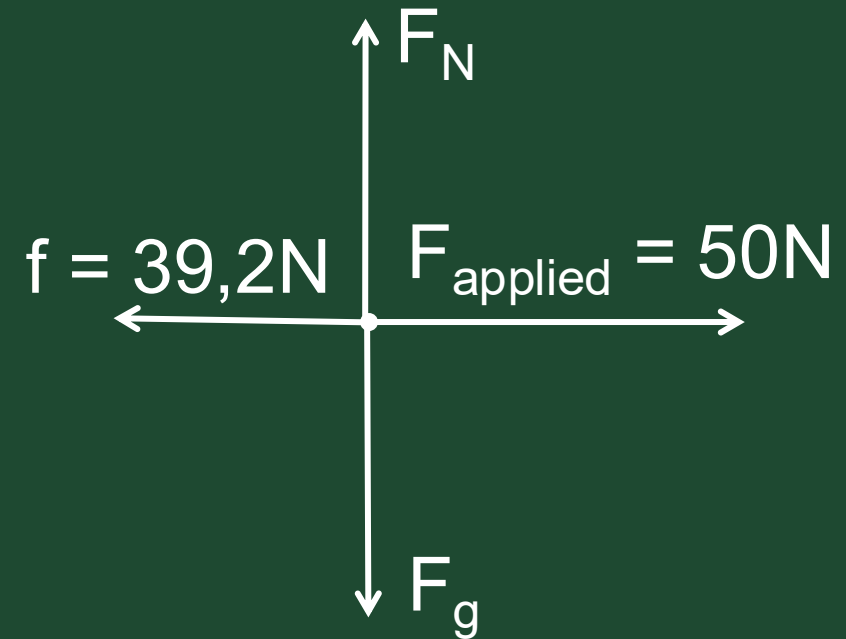


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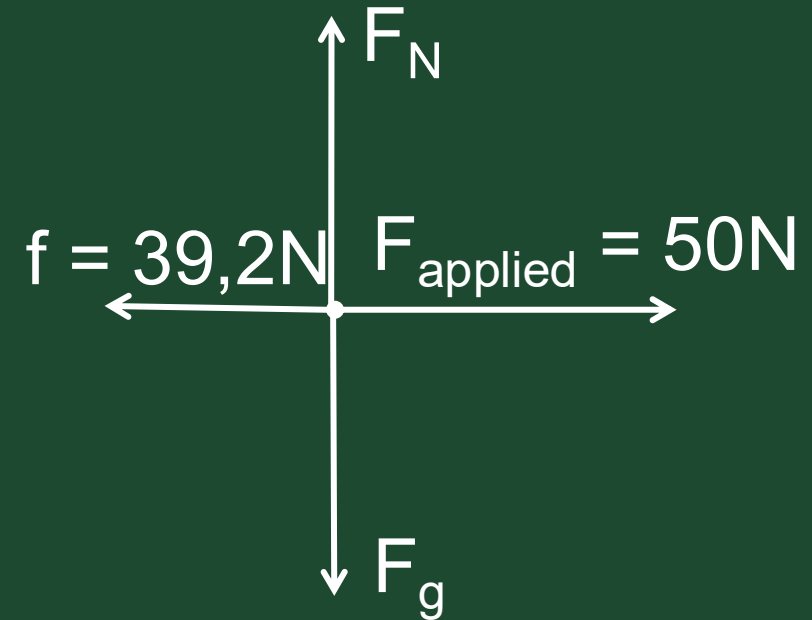
3.2 Calculate the work done on the cabinet by each of the forces.



# Example 3

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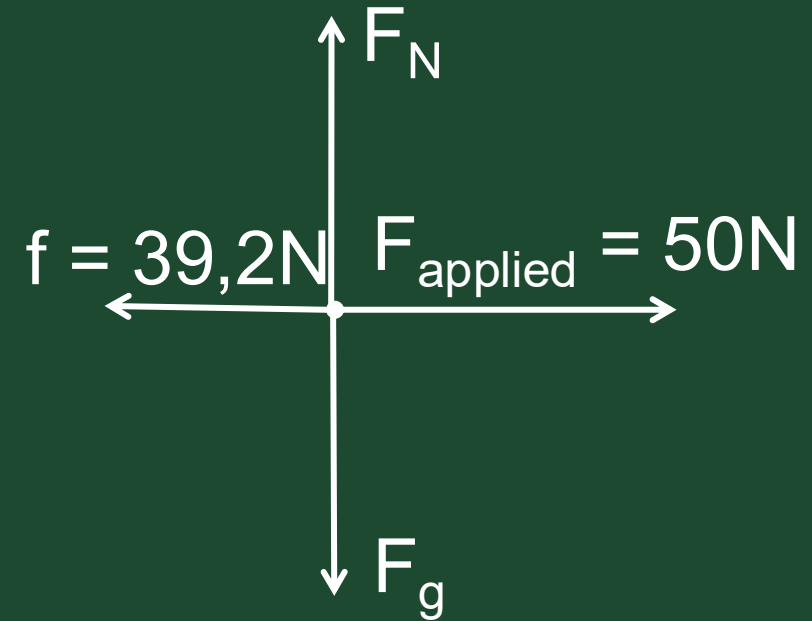
$$\begin{aligned}W_{F_g} &= F_g \Delta x \cos \theta \\ &= F_g \Delta x \cos 90^\circ \\ &= 0\text{J}\end{aligned}$$



# Example 3

3.2 Calculate the work done on the cabinet by each of the forces.

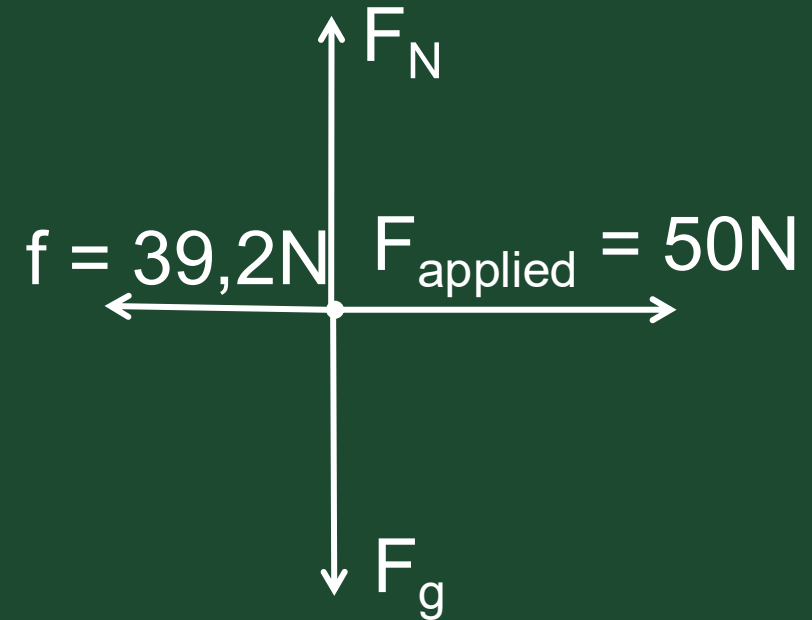
$$\begin{aligned}W_N &= N\Delta x \cos\theta \\ &= N\Delta x \cos 90^\circ \\ &= 0\text{J}\end{aligned}$$



# Example 3

3.2 Calculate the work done on the cabinet by each of the forces.

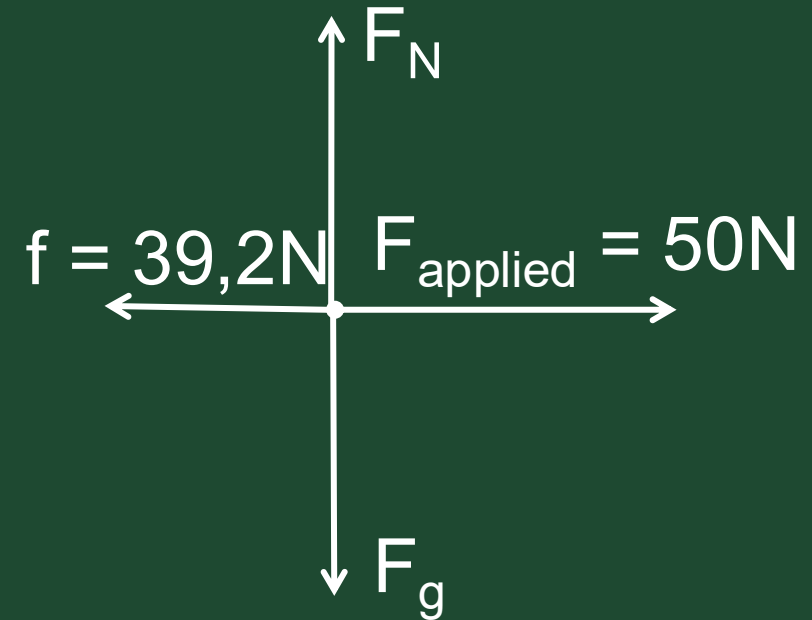
$$\begin{aligned}W_F &= F \Delta x \cos \theta \\&= 50 \times 5 \times \cos 0^\circ \\&= 250 \text{ J}\end{aligned}$$



# Example 3

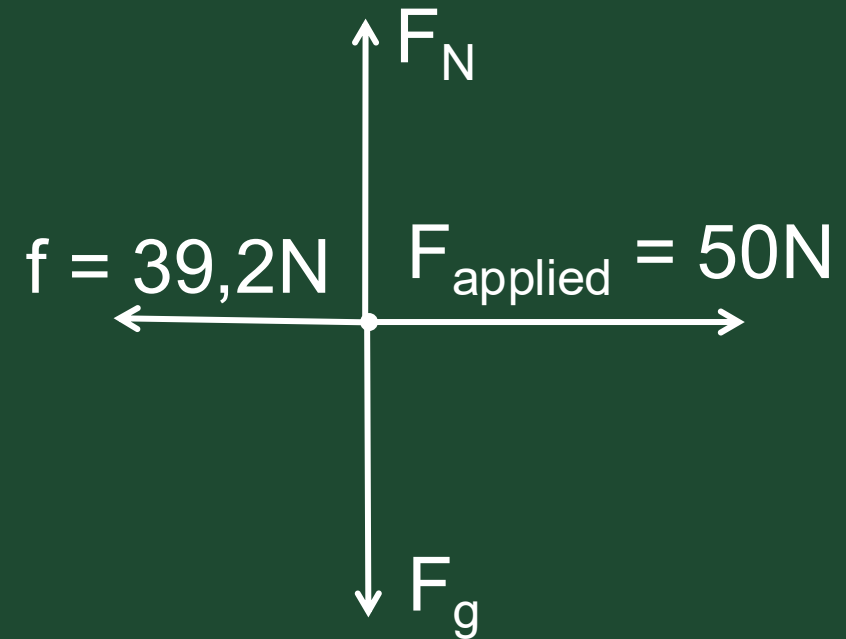
3.2 Calculate the work done on the cabinet by each of the forces.

$$\begin{aligned}W_f &= f\Delta x \cos\theta \\ &= 39,2 \times 5 \times \cos 180^\circ \\ &= -196\text{J}\end{aligned}$$



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A learner tries to move a heavy cabinet, with a mass of 80kg, by applying a 50N push force parallel to the floor on it. The cabinet experiences a frictional force of 39,2N. He manages to move the cabinet for a distance of 5m.

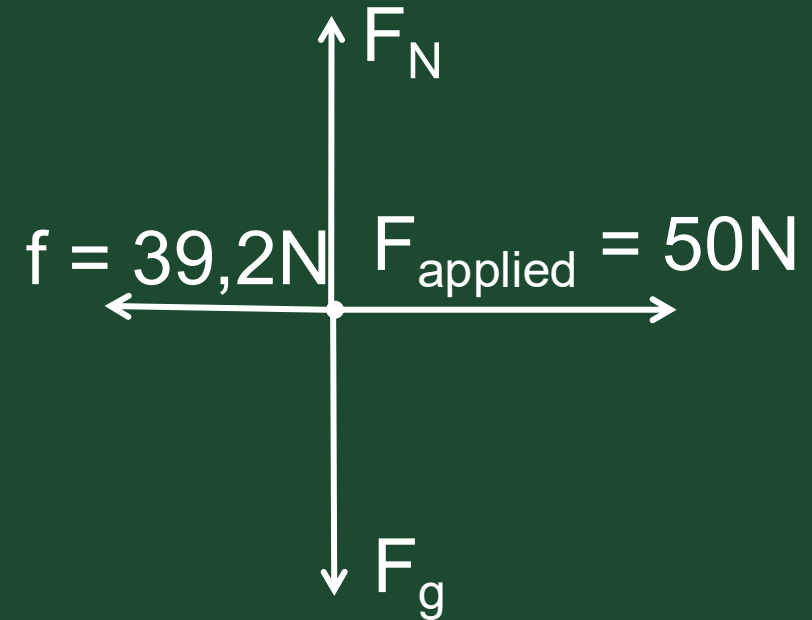


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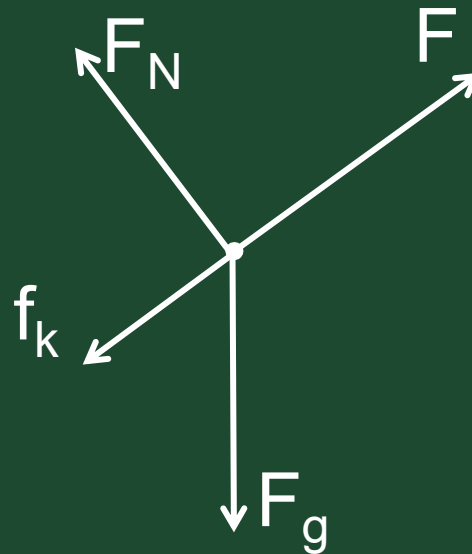
$$\begin{aligned}W_{\text{net}} &= F_{\text{net}} \Delta x \cos \theta \\ &= (50 - 39,2) (5) (\cos 0^\circ) \\ &= 54 \text{ J}\end{aligned}$$



# Example 4

A man pulls a cart with a load up a hill that forms an angle of  $25^\circ$  with the horizontal. The mass of the cart and load is 350kg and the frictional force of the road on the cart and load is 180N. He walks a distance of half a kilometre at a constant velocity (accept that the gradient remains unchanged).

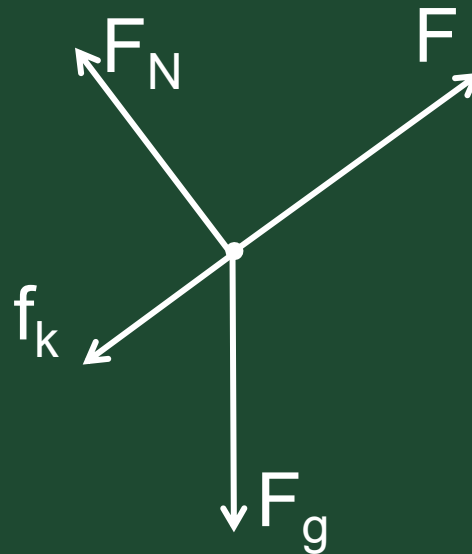
- 4.1 Draw a free-body diagram of all the forces acting on the cart with its load.



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4.2 Determine the net work done on the cart and load.

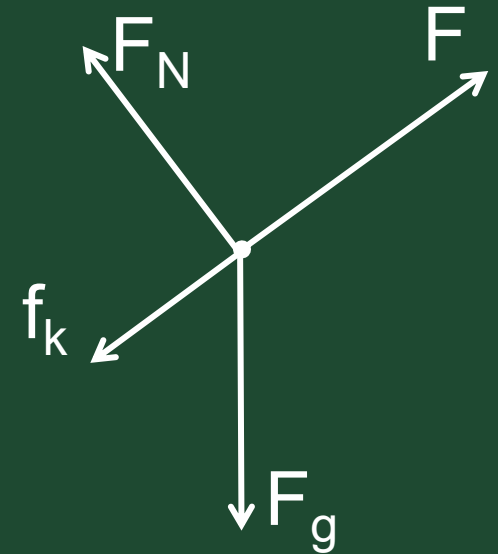


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4.2 Determine the net work done on the cart and load.

$$\begin{aligned}W_{\text{net}} &= F_{\text{net}} \Delta x \cos\theta \\ &= 0\text{J}\end{aligned}$$



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A man pulls a cart with a load up a hill that forms an angle of  $25^\circ$  with the horizontal. The mass of the cart and load is 350kg and the frictional force of the road on the cart and load is 180N. He walks a distance of half a kilometre at a constant velocity (accept that the gradient remains unchanged).

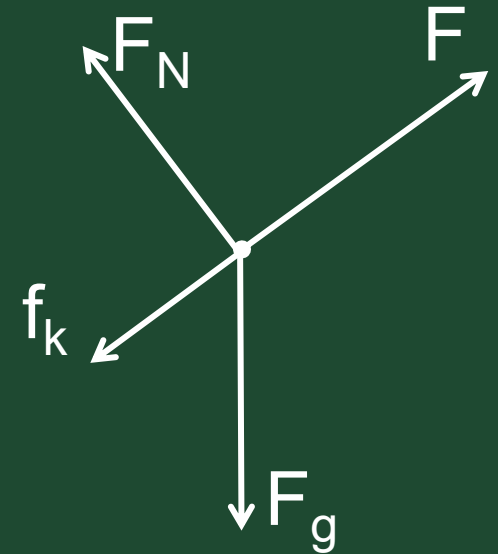
4.3 Calculate the work that the man does on the cart and the load.

$$F_{\text{net}} = F + F_{g//} + f = 0$$

$$F + (-350 \times 9,8 \times \sin 25^\circ) + (-180) = 0$$

$$F = 1629,58\text{N}$$

$$\begin{aligned} W_F &= F \Delta x \cos\theta \\ &= (1629,58)(500)(\cos 0^\circ) \\ &= 814790,32\text{J} \end{aligned}$$



# Work-energy theorem

The net/ total work done on an object is equal to the **CHANGE** in the object's kinetic energy

OR

The work done on an object by a resultant/ net force is equal to the change in the object's kinetic energy

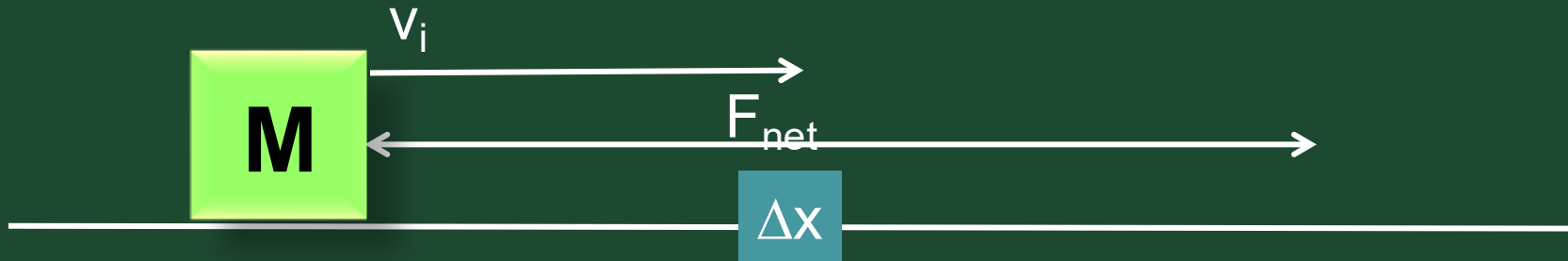


# Work-energy theorem

$$W_{\text{net}} = \Delta E_k$$



# Work-energy theorem



$$v_f^2 = v_i^2 + 2a\Delta x$$

$$a = \left( \frac{1}{2} v_f^2 - \frac{1}{2} v_i^2 \right) / \Delta x$$

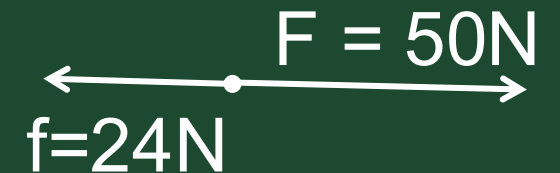
$$\begin{aligned} W_{net} &= F_{net} \Delta x \cos \theta \\ &= (ma) \Delta x \cos 0 \\ &= \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 \\ &= \Delta E_k \end{aligned}$$



# Example 5

A father pushes his son on a tricycle, by applying a horizontal force of 50N. The mass of the son and the tricycle is 35kg.

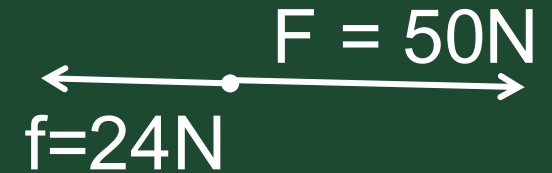
Calculate the velocity the son reaches on the tricycle after the father has pushed him a distance of 20m from a stationary position. The road exerts a frictional force of 8N on each wheel.



# Example 5

A father pushes his son on a tricycle, by applying a horizontal force of 50N. The mass of the son and the tricycle is 35kg.

Calculate the velocity the son reaches on the tricycle after the father has pushed him a distance of 20m from a stationary position. The road exerts a frictional force of 8N on each wheel.



$$\begin{aligned}W_{\text{net}} &= \Delta E_k \\(50 + (-24)) \times 20 \times \cos 0^\circ &= \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 \\520 &= \frac{1}{2} (35) v_f^2 - 0 \\\therefore v_f &= 5,45 \text{ m.s}^{-1}; \text{ in the direction of motion}\end{aligned}$$

# Conservative forces

A force for which the work done in moving an object between two points is independent of the path taken.

Example:

Gravitational force



# Non-conservative forces

A force for which the work done in moving an object between two points depends on the path taken.

Example:

Frictional force



# The principle of conservation of mechanical energy

The total mechanical energy in an isolated system remains constant.

NOTE: A system is isolated when the net external force (excluding the gravitational force) acting on the system is zero.

$$E_{M(A)} = E_{M(B)}$$



As you continue on your WEP journey, you will be introduced to the formulae below.

**WORK, ENERGY AND POWER**

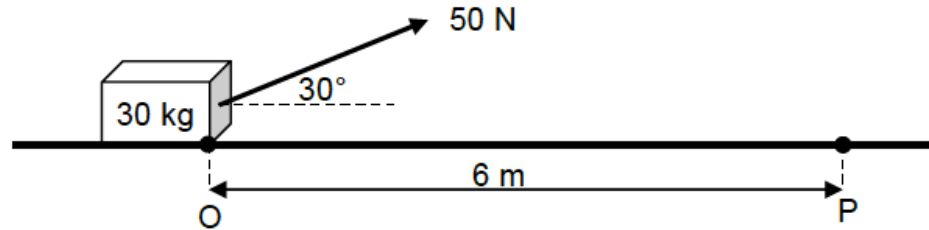
$W = F\Delta x \cos \theta$	$U = mgh$ OR $E_p = mgh$
$K = \frac{1}{2}mv^2$ OR $E_k = \frac{1}{2}mv^2$	$W_{\text{net}} = \Delta K$ OR $W_{\text{net}} = \Delta E_k$ $\Delta K = K_f - K_i$ OR $\Delta E_k = E_{kf} - E_{ki}$
$W_{\text{nc}} = \Delta K + \Delta U$ OR $W_{\text{nc}} = \Delta E_k + \Delta E_p$	$P = \frac{W}{\Delta t}$
$P_{\text{ave}} = Fv_{\text{ave}}$	



# Structured questions

## QUESTION 10

A worker pulls a crate of mass 30 kg from rest along a horizontal floor by applying a constant force of magnitude 50 N at an angle of  $30^\circ$  to the horizontal. A frictional force of magnitude 20 N acts on the crate whilst moving along the floor.



- 10.1 Draw a labelled free-body diagram to show ALL the forces acting on the crate during its motion.
- 10.2 Give a reason why each of the vertical forces acting on the crate do NO WORK on the crate.
- 10.3 Calculate the net work done on the crate as it reaches point P, 6 m from the starting point O.
- 10.4 Use the work-energy theorem to calculate the speed of the crate at the instant it reaches point P.
- 10.5 The worker now applies a force of the same magnitude, but at a SMALLER ANGLE to the horizontal, on the crate.

How does the work done by the worker now compare to the work done by the worker in QUESTION 10.3? Write down only GREATER THAN, SMALLER THAN or EQUAL TO.

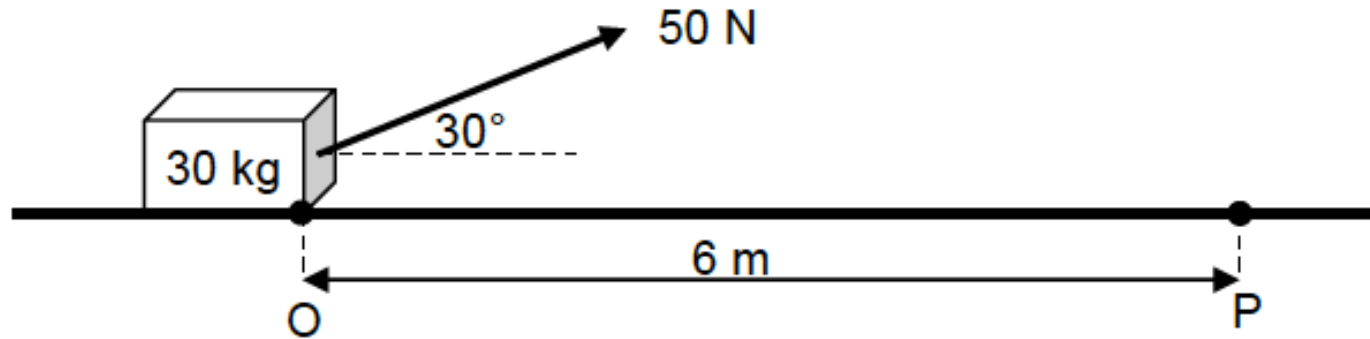
Give a reason for the answer. (No calculations are required.)



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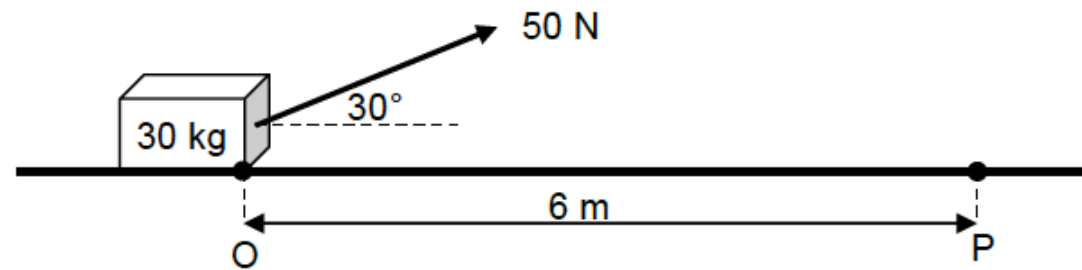
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A worker pulls a crate of mass 30 kg from rest along a horizontal floor by applying a constant force of magnitude 50 N at an angle of  $30^\circ$  to the horizontal. A frictional force of magnitude 20 N acts on the crate whilst moving along the floor.



$$m = 30 \text{ kg}$$

$$v_i = 0 \text{ m.s}^{-1}$$

$$F_A = 50 \text{ N (} 30^\circ \text{ to horizontal)}$$

$$\Delta x = 6 \text{ m (from O to P)}$$

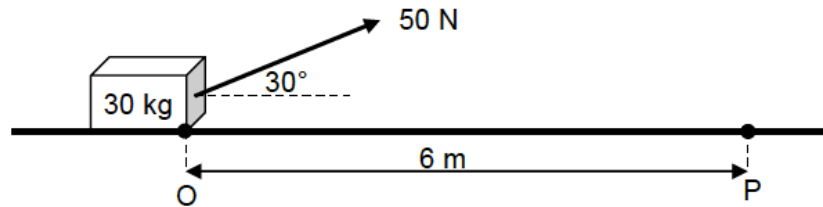
$$f = 20 \text{ N, left}$$



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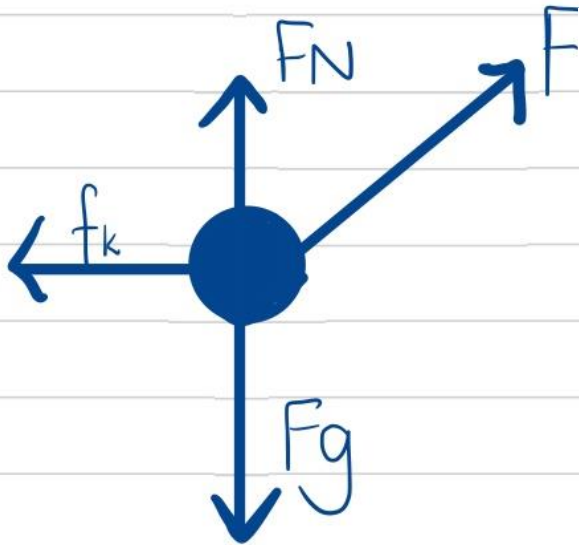
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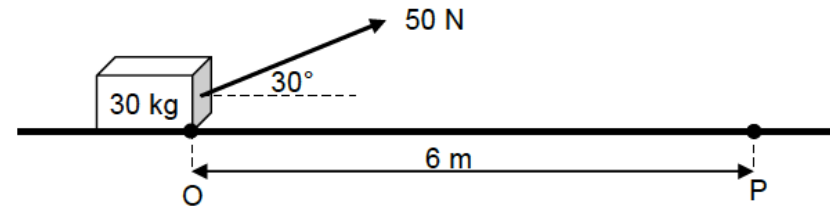
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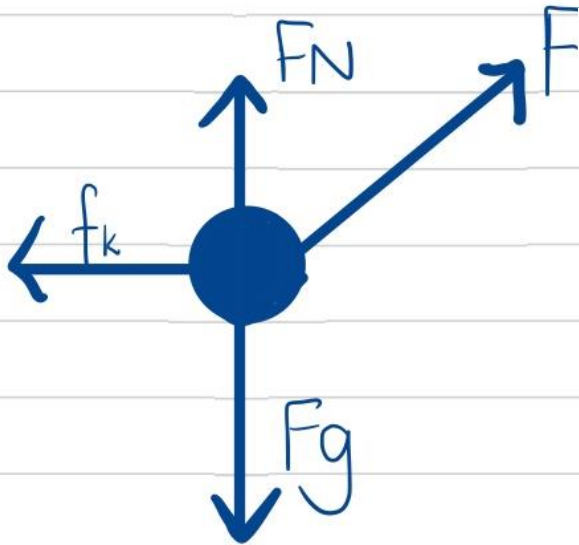
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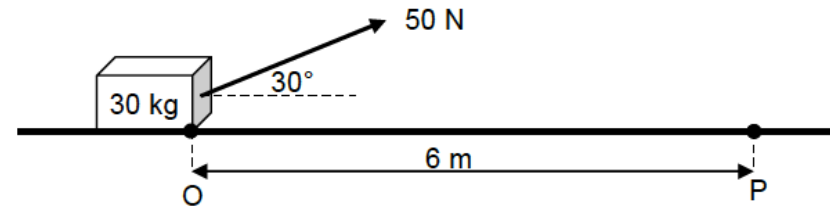


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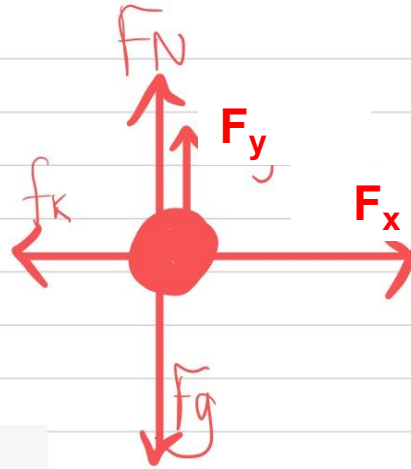
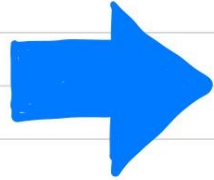
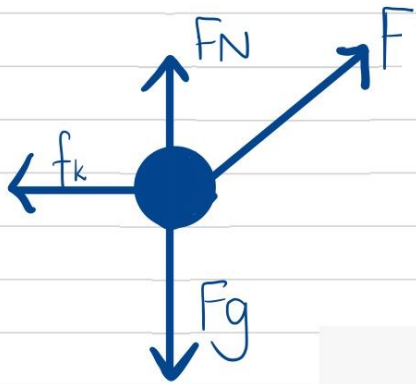
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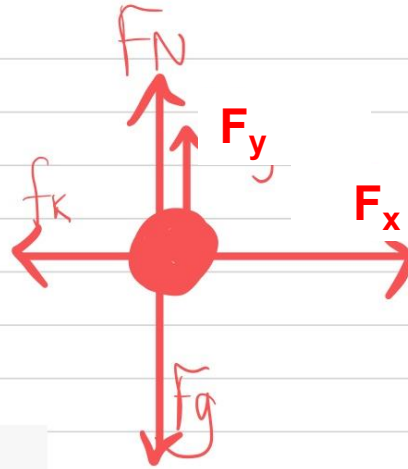
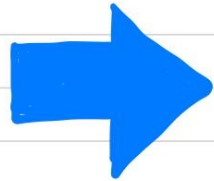
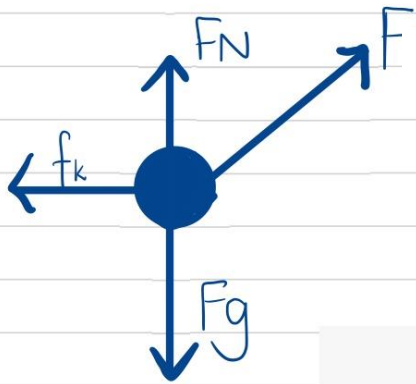
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- 10.2 Give a reason why each of the vertical forces acting on the crate do NO WORK on the crate.



# Structured questions



# Structured questions



Perpendicular to the direction of the displacement/ motion of the crate.

The forces act vertically; however, the crate is moving horizontally

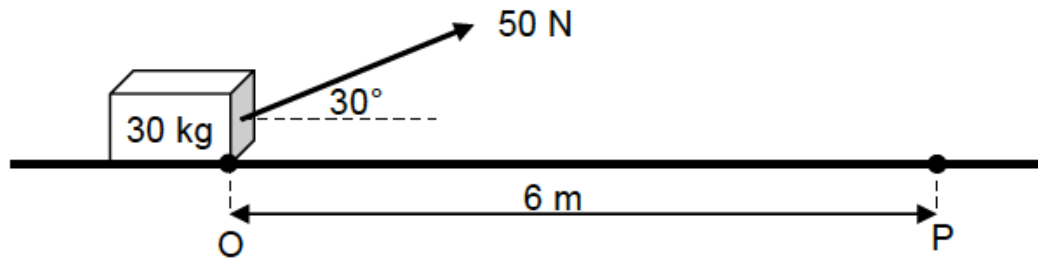
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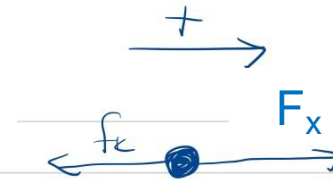
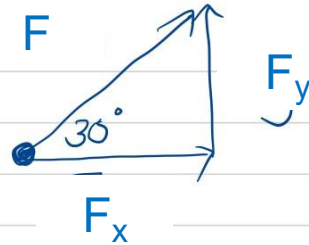
- 10.3 Calculate the net work done on the crate as it reaches point **P**, 6 m from the starting point **O**.



# Structured questions

10.3 Calculate the net work done on the crate as it reaches point P, 6 m from the starting point O.

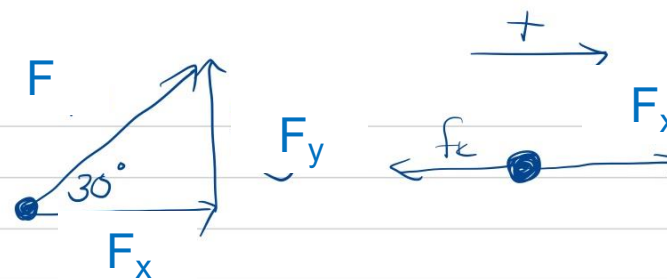
$$\begin{aligned}\vec{F}_x &= F \cos 30^\circ \\ &= 50 \cdot \cos 30^\circ \\ &= 43,301 \text{ N; right}\end{aligned}$$



# Structured questions

10.3 Calculate the net work done on the crate as it reaches point P, 6 m from the starting point O.

$$\begin{aligned} F_x &= F \cos 30^\circ \\ &= 50 \cdot \cos 30^\circ \\ &= 43,301 \text{ N; right} \end{aligned} \quad (1)$$

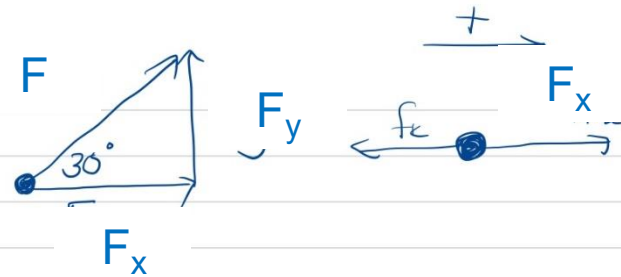


$$\begin{aligned} (2) \quad F_{\text{net}} &= F_{Ax} + (-f) \\ &= 43,301 - 20 \\ &= 23,301 \text{ N; right} \end{aligned}$$

# Structured questions

10.3 Calculate the net work done on the crate as it reaches point P, 6 m from the starting point O.

$$\begin{aligned} F_x &= F \cos 30^\circ \quad (1) \\ &= 50 \cdot \cos 30^\circ \\ &= 43,301 \text{ N; right} \end{aligned}$$



$$\begin{aligned} (2) \quad F_{\text{net}} &= F_x + (-f) \\ &= 43,301 - 20 \\ &= 23,301 \text{ N; right} \end{aligned}$$

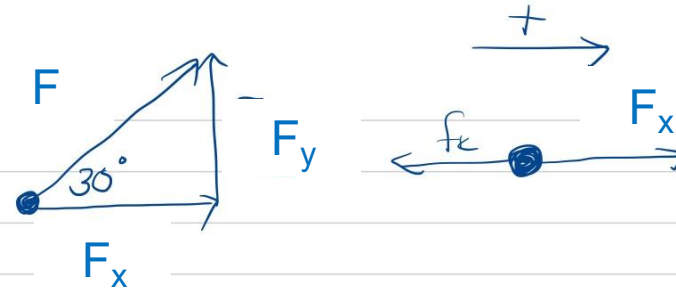


Remember!  $\Delta x = 6 \text{ m; right!}$

$\therefore \cos 0^\circ$

# Structured questions

$$\begin{aligned} F_x &= F \cos 30^\circ \quad (1) \\ &= 50 \cdot \cos 30^\circ \\ &= 43,301 \text{ N; right} \end{aligned}$$



$$\begin{aligned} (2) \quad F_{\text{net}} &= F_x + (-f) \\ &= 43,301 - 20 \\ &= 23,301 \text{ N; right} \end{aligned}$$

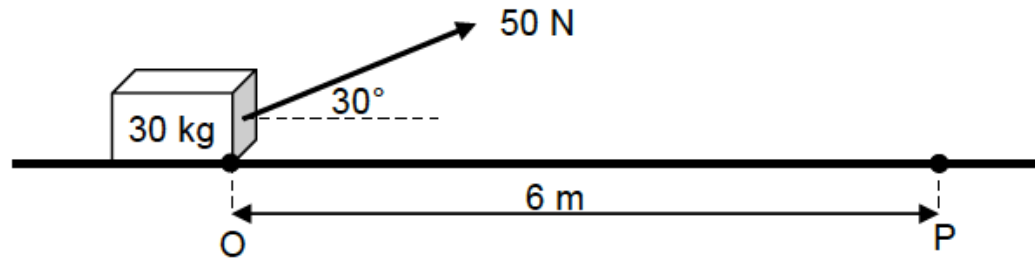


$$\begin{aligned} (3) \quad W_{\text{net}} &= F_{\text{net}} \Delta x \cos \theta \\ &= (23,301) (6) (\cos 0^\circ) \\ &= 139,81 \text{ J} \end{aligned}$$

# Structured questions

## QUESTION 10

A worker pulls a crate of mass 30 kg from rest along a horizontal floor by applying a constant force of magnitude 50 N at an angle of  $30^\circ$  to the horizontal. A frictional force of magnitude 20 N acts on the crate whilst moving along the floor.



- 10.4 Use the work-energy theorem to calculate the speed of the crate at the instant it reaches point P.

# Structured questions

10.4 Use the work-energy theorem to calculate the speed of the crate at the instant it reaches point **P**.

$$W_{\text{net}} = \Delta E_k$$

$$v_f = ?$$

$$v_i = 0 \text{ m}\cdot\text{s}^{-1}$$

$$m = 30 \text{ kg}$$

$$W_{\text{net}} = 139,81 \text{ J}$$



# Structured questions

10.4 Use the work-energy theorem to calculate the speed of the crate at the instant it reaches point **P**.

$$W_{\text{net}} = \Delta E_k$$
$$W_{\text{net}} = E_{kf} - E_{ki}$$
$$W_{\text{net}} = \left(\frac{1}{2}mv_f^2\right) - \left(\frac{1}{2}mv_i^2\right)$$
$$139,81 = \left(\frac{1}{2} \times 30 \times v_f^2\right) - \left(\frac{1}{2} \times 30 \times 0^2\right)$$
$$139,81 = \left(\frac{1}{2} \times 30 \times v_f^2\right) - 0$$
$$\therefore v_f = 3,05 \text{ m}\cdot\text{s}^{-1}$$
$$\therefore \text{the speed will be } 3,05 \text{ m}\cdot\text{s}^{-1}$$

$$v_f = ?$$

$$v_i = 0 \text{ m}\cdot\text{s}^{-1}$$

$$m = 30 \text{ kg}$$

$$W_{\text{net}} = 139,81 \text{ J}$$



# Structured questions

10.5 The worker now applies a force of the same magnitude, but at a SMALLER ANGLE to the horizontal, on the crate.

How does the work done by the worker now compare to the work done by the worker in QUESTION 10.3? Write down only GREATER THAN, SMALLER THAN or EQUAL TO.

Give a reason for the answer. (No calculations are required.)



# Structured questions

10.5 The worker now applies a force of the same magnitude, but at a SMALLER ANGLE to the horizontal, on the crate.

How does the work done by the worker now compare to the work done by the worker in QUESTION 10.3? Write down only GREATER THAN, SMALLER THAN or EQUAL TO.

Give a reason for the answer. (No calculations are required.)

GREATER THAN

The horizontal component of the applied force in the direction of the motion will be greater if  $\theta$  decreases.

Therefore from  $W_{\text{horizontal component}} = F_{Ax} \Delta x \cos\theta$  it is clear that the work done by the worker will be greater.

